## International Journal of Maritime Engineering

# ON THE EFFECT OF TANK FREE SURFACES ON VESSEL STATIC STABILITY 

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## SUMMARY

This paper examines the effect of fluid free surfaces in slack tanks on a vessel's transverse, static stability. The exact, transverse movement of fluid in a half-full, rectangular tank and its effect on vessel stability is derived. This is compared with results from computer software that also models the static position of the fluid in the tank, including the effects of both heel and trim; the traditional correction to vertical centre of gravity based on upright second moment of area of the tank waterplane; and the IMO free surface moment method as described in IMO A.749(18) and MSC.75(69)[1, 2, §3.3]. Once validated, the application of the computer software has been demonstrated by using it to examine the free surface effect of tanks at filling conditions other than $50 \%$.

## NOMENCLATURE

| A | Area m ${ }^{2}$ |
| :---: | :---: |
| $b$ | Tank breadth m |
| CG | Centre of gravity |
| FSM | Free surface "moment" kg m |
| GM | Separation of metacentre and centre of gravity (ship axis system) m |
| GZ | Righting lever (arm) m |
| $I$ | Transverse $2^{\text {nd }}$ moment of area about area centroid $\mathrm{m}^{4}$ |
| $h$ | Tank height m |
| $T$ | Transverse lever (ship axis system) m |
| V | Vertical lever (ship axis system) m |
| $\delta y$ | Horizontal shift of tank CG (earth-fixed axis system) m |
| $\delta y_{\text {Cond.A }}$ | Horizontal shift of tank CG for Condition A (earth-fixed axis system) m |
| $\delta y_{s}$ | Horizontal shift of ship CG (earth-fixed axis system) m |
| $\phi$ | Heel angle rad (or ${ }^{\circ}$ ) |
| $\nabla$ | Volume of displacement $\mathrm{m}^{3}$ |
| $\rho$ | Fluid density $\mathrm{kg} \mathrm{m}^{-3}$ |
| subscripts |  |
| tri | triangle |
| rec | rectangle |
| imm | immersed |
| em | emerged |
| $s$ | ship |
| $t$ | tank |

## 1 INTRODUCTION

In this paper, the static effect of fluid shift due to heel in a half-filled, rectangular cross-section, prismatic tank is derived theoretically. These results are compared with the commonly used method of raising the vessel's effective
centre of gravity based on the upright, transverse second moment of inertia of the tank's waterplane. Depending on the tank's aspect ratio, the actual effect of the fluid shift in the tank can be significantly different from that estimated using this method.

The theoretical results are then used to validate a software program, Hydromax[3], which calculates the actual, static, position of the fluid in the tank, taking into account the effects of both heel and trim.

An example of the type of investigation which can be made using this type of software is included. In this paper, the effect of tank loading is investigated, but the effect of tank shape or other parameters could also be investigated.

Finally and perhaps most importantly, the IMO free surface correction is examined. It is shown that the equations specified in the IMO Code on Intact Stability[1, 2, §3.3] are derived from the same theoretical analysis as that presented in this paper, i.e.: the effect of a half-filled, rectangular cross-section, prismatic tank. It is also shown that the IMO equations are not valid for the complete range of heel angles and also a different interpretation of "free surface moment" is used by IMO. In the IMO code, these limitations are not highlighted and may not be apparent to the practitioner unless a similar analysis as that presented here is undertaken.

Note that this work assumes a quasi-static condition and no dynamic effects such as sloshing are considered.

## 2 FLUID SHIFT IN HALF-FULL, RECTANGULAR TANK

For the sake of simplicity we shall consider a prismatic tank, with rectangular cross-section, half-full of fluid. We shall also assume that the vessel is fixed in trim and hence look at the transverse cross-section only.

There are three conditions depending on the heel angle of the vessel. The two main conditions are illustrated in Fig-
ure 1 and the remaining condition is the inverted case of Condition A. Condition A is true for the heel angle range given by Equation 1; Condition A Inverted is valid for the heel angle range given by Equation 2 and Condition B is true for the middle heel angle range given by Equation 3.

## Condition A (moderate angle of heel)

$$
\begin{equation*}
0 \leq \phi \leq \tan ^{-1}\left(\frac{h}{b}\right) \tag{1}
\end{equation*}
$$

## Condition A inverted

$$
\begin{equation*}
\pi-\tan ^{-1}\left(\frac{h}{b}\right) \leq \phi \leq \pi \tag{2}
\end{equation*}
$$

## Condition B (increased angle of heel)

$$
\begin{equation*}
\tan ^{-1}\left(\frac{h}{b}\right) \leq \phi \leq \pi-\tan ^{-1}\left(\frac{h}{b}\right) \tag{3}
\end{equation*}
$$

In the derivation presented below, the shift of the tank centre of gravity $(\mathrm{CG})$ is calculated in the ship axis system, the horizontal shift of the tank CG in the earth-fixed axis system is then easily found by simply rotating the coordinate system.

### 2.1 CONDITION A

Consider the situation as shown in Figure 2, where the fluid in the tank has moved due to the vessel heeling to an angle $\phi$. The area of the immersed and emerged triangles is given by Equation 4

$$
\begin{equation*}
A=\frac{1}{2} \times \frac{b}{2} \times \frac{b \tan \phi}{2}=\frac{b^{2} \tan \phi}{8} \tag{4}
\end{equation*}
$$

We shall now find the shift of the fluid CG in the ship coordinate system. The transverse and vertical components of the centroid of the immersed triangle, measured from the upright tank CG (in the ship axis system), are given by Equations 5 and 6 respectively.

$$
\begin{gather*}
T_{\mathrm{imm}}=\frac{2}{3} \times \frac{b}{2}=\frac{b}{3}  \tag{5}\\
V_{\mathrm{imm}}=\frac{h}{2}+\frac{1}{3} \times \frac{b \tan \phi}{2}=\frac{h}{2}+\frac{b \tan \phi}{6} \tag{6}
\end{gather*}
$$

Similarly, the centroid of the emerged triangle is given by Equations 7 and 8 respectively.

$$
\begin{gather*}
T_{\mathrm{em}}=\frac{2}{3} \times \frac{-b}{2}=\frac{-b}{3}  \tag{7}\\
V_{\mathrm{em}}=\frac{h}{2}-\times \frac{1}{3} \frac{b \tan \phi}{2}=\frac{h}{2}-\frac{b \tan \phi}{6} \tag{8}
\end{gather*}
$$

The CG of the fluid in the heeled tank, in the ship coordinate system, can be found by taking moments about the original upright tank CG - Equations 9 and 10 .

$$
\begin{align*}
T \frac{b h}{2}= & -A \times T_{\mathrm{em}}+A \times T_{\mathrm{imm}} \\
T \frac{b h}{2}= & \frac{-b^{2} \tan \phi}{8} \times \frac{-b}{3}+\frac{b^{2} \tan \phi}{8} \times \frac{b}{3} \\
T= & \frac{b^{2} \tan \phi}{6 h}  \tag{9}\\
V \frac{b h}{2}= & -A \times V_{\mathrm{em}}+A \times V_{\mathrm{imm}} \\
V \frac{b h}{2}= & \frac{-b^{2} \tan \phi}{8} \times\left(\frac{h}{2}-\frac{b \tan \phi}{6}\right) \\
& +\frac{b^{2} \tan \phi}{8} \times\left(\frac{h}{2}+\frac{b \tan \phi}{6}\right) \\
V= & \frac{b^{2} \tan ^{2} \phi}{12 h} \tag{10}
\end{align*}
$$

The reduction of GZ, $\delta \mathrm{GZ}$, which is equal to the horizontal shift of the vessel CG in the earth-fixed axis system, $\delta y_{s}$, can be calculated from the horizontal shift of the tank CG in the earth-fixed axis system, $\delta y$, by taking moments about the original vessel CG - as shown in Equation 11.

$$
\begin{align*}
\delta y_{s} \nabla_{s} \rho_{s} & =\delta y \nabla_{t} \rho_{t} \\
\delta y_{s}=\delta \mathrm{GZ} & =\delta y \frac{\nabla_{t} \rho_{t}}{\nabla_{s} \rho_{s}} \tag{11}
\end{align*}
$$

An equivalent free surface moment, FSM, can be calculated by considering the effect of the tank CG shift to be equivalent to a virtual rise in CG (this is the traditional approach). In this case, $\delta \mathrm{GZ}=\delta \mathrm{VCG} \sin \phi$ and the virtual rise in the CG is given by Equation 12.

$$
\begin{equation*}
\delta \mathrm{VCG}=\frac{\mathrm{FSM}}{\nabla_{s} \rho_{s}} \tag{12}
\end{equation*}
$$

Hence

$$
\begin{equation*}
\delta \mathrm{GZ}=\frac{\mathrm{FSM}}{\nabla_{s} \rho_{s}} \sin \phi \tag{13}
\end{equation*}
$$

Rearranging Equation 13, the equivalent FSM can be calculated from Equation 14:

$$
\begin{equation*}
\mathrm{FSM}=\frac{\delta \mathrm{GZ} \nabla_{s} \rho_{s}}{\sin \phi} \tag{14}
\end{equation*}
$$

Substituting $\delta \mathrm{GZ}$ from Equation 11 into Equation 14 relates the horizontal shift of tank CG $(\delta y)$ to the equivalent FSM - Equation 15.

$$
\begin{equation*}
\mathrm{FSM}=\frac{\delta y \nabla_{t} \rho_{t}}{\sin \phi} \tag{15}
\end{equation*}
$$

Now the horizontal shift of the tank CG in the earth-fixed axis system is given by Equation 16.

$$
\begin{align*}
\delta y & =T \cos \phi+V \sin \phi \\
\delta y & =\frac{b^{2} \tan \phi}{6 h} \cos \phi+\frac{b^{2} \tan ^{2} \phi}{12 h} \sin \phi \tag{16}
\end{align*}
$$



Figure 1: Two conditions arise depending on the heel angle


Figure 2: Shift of fluid for $0 \leq \phi \leq \tan ^{-1}(h / b)$

Thus substituting for $\delta y$, Equation 16, into Equation 15 yields the equivalent FSM - Equation 17 which simplifies to Equation 18.

$$
\begin{equation*}
\mathrm{FSM}=\left(\frac{b^{2} \tan \phi}{6 h} \cos \phi+\frac{b^{2} \tan ^{2} \phi}{12 h} \sin \phi\right) \frac{\nabla_{t} \rho_{t}}{\sin \phi} \tag{17}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{FSM}=\frac{\nabla_{t} \rho_{t} b^{2}}{6 h}\left(1+\frac{\tan ^{2} \phi}{2}\right) \tag{18}
\end{equation*}
$$

### 2.2 CONDITION A INVERTED

In this condition, it may be shown that the horizontal shift in tank CG, in the earth-fixed axis system is, given by Equation 19:

$$
\begin{align*}
\delta y & =\frac{h}{2} \sin \phi-\delta y_{\text {Cond.A }} \\
\delta y & =\frac{h}{2} \sin \phi-\frac{b^{2} \tan \phi}{6 h} \cos \phi-\frac{b^{2} \tan ^{2} \phi}{12 h} \sin \phi \tag{19}
\end{align*}
$$

where $\delta y_{\text {Cond.A }}$ is the earth-fixed, horizontal shift for Condition A - Equation 18.

Substituting for $\delta y$, Equation 19, into Equation 15 and simplifying, yields the equivalent FSM - Equation 20.

$$
\begin{equation*}
\mathrm{FSM}=\frac{\nabla_{t} \rho_{t} b^{2}}{6 h}\left(\frac{3 h^{2}}{b^{2}}-1-\frac{\tan ^{2} \phi}{2}\right) \tag{20}
\end{equation*}
$$

### 2.3 CONDITION B

Now we shall consider the situation as shown in Figure 3, where the heel angle has been increased. The immersed and emerged areas can be divided into a triangle and a rectangle. Considering the triangles first: the area of the immersed and emerged triangles is given by Equation 21.

$$
\begin{equation*}
A_{\mathrm{tri}}=\frac{1}{2} \times \frac{h}{2} \times \frac{h}{2 \tan \phi}=\frac{h^{2}}{8 \tan \phi} \tag{21}
\end{equation*}
$$

The transverse and vertical components of the centroid of immersed triangle are given by Equations 22 and 23 respectively.

$$
\begin{gather*}
T_{\text {tri imm }}=\frac{2}{3} \times \frac{h}{2 \tan \phi}=\frac{h}{3 \tan \phi}  \tag{22}\\
V_{\text {tri imm }}=\frac{h}{4}+\frac{1}{3} \times \frac{h}{2}=\frac{5 h}{12} \tag{23}
\end{gather*}
$$



Figure 3: Shift of fluid at $\tan ^{-1}(h / b)<\phi<2 \pi-\tan ^{-1}(h / b)$

Similarly, the centroid of the emerged triangle is given by Equations 24 and 25 respectively.

$$
\begin{gather*}
T_{\text {tri em }}=\frac{2}{3} \times \frac{-h}{2 \tan \phi}=\frac{-h}{3 \tan \phi}  \tag{24}\\
V_{\text {tri em }}=\frac{-h}{4}+\frac{2}{3} \times \frac{h}{2}=\frac{h}{12} \tag{25}
\end{gather*}
$$

Now consider the rectangles: the area of the immersed and emerged rectangles is given by Equation 26.

$$
\begin{equation*}
A_{\mathrm{rec}}=\frac{1}{2} \times \frac{h}{2} \times\left(b-\frac{h}{\tan \phi}\right)=\frac{1}{4}\left(b h-\frac{h^{2}}{\tan \phi}\right) \tag{26}
\end{equation*}
$$

The transverse and vertical components of the centroid of immersed rectangle are given by Equations 27 and 28 respectively.

$$
\begin{align*}
T_{\text {rec imm }}= & \frac{h}{2 \tan \phi}+\frac{1}{4}\left(b-\frac{h}{\tan \phi}\right) \\
= & \frac{1}{4}\left(b+\frac{h}{\tan \phi}\right)  \tag{27}\\
& V_{\text {rec imm }}=\frac{h}{2} \tag{28}
\end{align*}
$$

Similarly, the centroid of the emerged rectangle is given by Equations 29 and 30 respectively.

$$
\begin{align*}
T_{\mathrm{rec} \mathrm{em}}= & \frac{-h}{2 \tan \phi}-\frac{1}{4}\left(b-\frac{h}{\tan \phi}\right) \\
= & \frac{-1}{4}\left(b+\frac{h}{\tan \phi}\right)  \tag{29}\\
& V_{\mathrm{rec} \mathrm{em}}=0 \tag{30}
\end{align*}
$$

The CG of the fluid in the heeled tank, in the ship coordinate system, can be found by taking moments about the upright CG - Equations 31 and 32

$$
\begin{align*}
T \frac{b h}{2}= & -A_{\text {tri }} \times T_{\text {tri em }}-A_{\text {rec }} \times T_{\text {rec em }} \\
& +A_{\text {tri }} \times T_{\text {tri imm }}+A_{\text {rec }} \times T_{\text {rec imm }} \\
T \frac{b h}{2}= & -\frac{h^{2}}{8 \tan \phi} \times \frac{-h}{3 \tan \phi} \\
& -\frac{1}{4}\left(b h-\frac{h^{2}}{\tan \phi}\right) \times \frac{-1}{4}\left(b+\frac{h}{\tan \phi}\right) \\
& +\frac{h^{2}}{8 \tan \phi} \times \frac{h}{3 \tan \phi} \\
& +\frac{1}{4}\left(b h-\frac{h^{2}}{\tan \phi}\right) \times \frac{1}{4}\left(b+\frac{h}{\tan \phi}\right) \\
T= & \frac{b}{4}-\frac{h^{2}}{12 b \tan ^{2} \phi} \tag{31}
\end{align*}
$$

$$
\begin{align*}
V \frac{b h}{2}= & -A_{\text {tri }} \times V_{\text {tri em }}-A_{\text {rec }} \times V_{\text {rec em }} \\
& +A_{\text {tri }} \times V_{\text {tri imm }}+A_{\text {rec }} \times V_{\text {rec imm }} \\
V \frac{b h}{2}= & -\frac{h^{2}}{8 \tan \phi} \times \frac{h}{12}-\frac{1}{4}\left(b h-\frac{h^{2}}{\tan \phi}\right) \times 0 \\
& +\frac{h^{2}}{8 \tan \phi} \times \frac{5 h}{12}+\frac{1}{4}\left(b h-\frac{h^{2}}{\tan \phi}\right) \times \frac{h}{2} \\
V= & \frac{h}{4}-\frac{h^{2}}{6 b \tan \phi} \tag{32}
\end{align*}
$$

Again the horizontal shift of the tank CG in the earth-fixed


Figure 4: Comparison of theoretical effective FSM with Hydromax fluid simulation, expressed as percentage of FSM calculated from the upright tank waterplane.
axis system is given by Equation 33:

$$
\begin{align*}
\delta y= & T \cos \phi+V \sin \phi \\
\delta y= & \left(\frac{b}{4}-\frac{h^{2}}{12 b \tan ^{2} \phi}\right) \cos \phi \\
& +\left(\frac{h}{4}-\frac{h^{2}}{6 b \tan \phi}\right) \sin \phi \tag{33}
\end{align*}
$$

and hence the equivalent FSM is given by Equation 34, which simplifies to Equation 35

$$
\begin{align*}
\mathrm{FSM}= & \frac{\delta y \nabla_{t} \rho_{t}}{\sin \phi} \\
\mathrm{FSM}= & {\left[\left(\frac{b}{4}-\frac{h^{2}}{12 b \tan ^{2} \phi}\right) \cos \phi\right.} \\
& \left.+\left(\frac{h}{4}-\frac{h^{2}}{6 b \tan \phi}\right) \sin \phi\right] \frac{\nabla_{t} \rho_{t}}{\sin \phi} \tag{34}
\end{align*}
$$

$\mathrm{FSM}=\nabla_{t} \rho_{t}\left[\frac{h}{4}+\frac{b}{4 \tan \phi}-\frac{h^{2}}{6 b \tan \phi}\left(1+\frac{1}{2 \tan ^{2} \phi}\right)\right]$

## 3 STATIC SIMULATION OF FLUID

The equations derived above hold only for prismatic tanks with rectangular cross-section, half-filled with fluid. Although similar equations could be derived for other crosssection shapes, this is rather a cumbersome approach. With a computer model, it is relatively easy to compute the actual position of the fluid in the tank as the vessel heels and trims. This calculation method is available in Formation Design Systems' Hydromax stability software. Using this (or similar software) it is possible to investigate arbitrarily shaped tanks, that are not necessarily prismatic, with any level of fluid. Figure 4 shows a comparison of the theoretical FSM derived in this paper with


Figure 5: Effect of tank fluid volume on effective FSM, expressed as percentage of FSM calculated from the upright tank waterplane, for a $1 \mathrm{~m} \times 1 \mathrm{~m} \times 1 \mathrm{~m}$ cubic tank.


Figure 6: Effect of tank breadth:height ratio $(b / h)$ on effective FSM (broad tanks), expressed as percentage of FSM calculated from the upright tank waterplane.
the Hydromax fluid simulation. The results are presented for two prismatic tanks, one having a square cross-section ( $b / h=1$ ) and the other having a rectangular cross-section ( $b / h=1.722$ ). Results are expressed as a percentage of the FSM as computed from the upright waterplane $\left(I_{t} \rho_{t}\right)$ and as expected tend to $100 \%$ of the upright FSM at zero heel.

Hydromax has then been used to investigate the effect of tank fluid level on the FSM in the following section.

### 3.1 EFFECT OF VOLUME OF FLUID IN TANK

The traditional FSM based on the upright tank geometry is given by Equation 36

$$
\begin{equation*}
\mathrm{FSM}_{\text {upright }}=I_{t} \rho_{t} \tag{36}
\end{equation*}
$$



Figure 7: Effect of tank breadth:height ratio $(b / h)$ on effective FSM (tall tanks), expressed as percentage of FSM calculated from the upright tank waterplane.
where $I_{t}$ is the transverse second moment of area of tank waterplane, about its centroid, for the upright tank and $\rho_{t}$ is the density of the fluid in the tank.

It can be seen that Equation 36 is independent of the volume of fluid in the tank. However, in reality this is not the case and the effective FSM does depend on the volume of fluid in the tank. Figure 5 shows the actual effect of the fluid movement in the tank as an effective FSM for different levels of fluid in the tank. (The same effect is observed if the tank is $x \%$ full or $100-x \%$ full.) It can be seen that the maximum effect occurs when the tank is half-full ( $50 \%$ ). These results were computed for a $1 \mathrm{~m} \times 1 \mathrm{~m} \times 1 \mathrm{~m}$ cubic tank.

### 3.2 EFFECT OF TANK BREADTH:HEIGHT RATIO

The effect of tank breadth:height ratio $(b / h)$ on FSM has been investigated and the results presented in Figures 6 and 7. The values are expressed as a percentage of the FSM calculated from the upright waterplane. These results are for a half-full tank with rectangular cross-section.

The results for tanks with, $b / h \geq 1.0$, tanks are shown in Figure 6. For the broader tanks, $b / h>3.0$, the effective FSM reduces rapidly compared with the upright FSM at heel angles above $30^{\circ}$.

It can be seen that there is a dramatic effect for tall, narrow tanks (Figure7), especially at large heel angles. This could be an important consideration for self-righting craft or craft that are expected to operate in severe conditions. In some situations, where large angles of vessel heel are expected, the case could be made for broad tanks, which despite reducing initial stability would be less detrimental as heel angle increases.

However, perhaps a more useful comparison can be made by examining the actual effective FSM for tanks of dif-
ferent breadth:height ratio but constant total capacity; this has been done and the results are shown in Figure 8. In this case the broad tanks have significantly higher FSM upright, but the FSM diminishes quite rapidly as heel angle increases and becomes negative once the vessel is heeled above approximately $100^{\circ}$. The narrow tanks, on the other hand, have a reduced FSM in the upright condition, but the FSM increases with heel angle, with a significant jump at approximately $90^{\circ}$. Hence at heel angles above about $80^{\circ}$, the broad tanks have reduced FSM compared with the narrow tanks (for the same capacity).

## 4 COMPARISON WITH IMO

In $\S 3.3$, the IMO code on intact stability[1], amended by [2], describes how fluid free surfaces in slack tanks should be taken into account, the relevant section of the code is reproduced below:

### 3.3 Effect of free surfaces of liquids in tanks

3.8 The values of $M_{\mathrm{fs}}$ for each tank may be derived from the formula:

$$
M_{\mathrm{fs}}=v b \rho h k \sqrt{\delta}
$$

where:
$M_{\mathrm{fs}}$ is the free surface moment at any inclination, in m tonnes
$v$ is the total tank capacity, in $\mathrm{m}^{3}$
$b$ is the tank maximum breadth, in $m$
$\rho$ is the mass density of liquid in the tank, in tonnes $/ \mathrm{m}^{3}$
$\delta$ is equal to $v / b l h$ (the tank block coefficient)
$h$ is the tank maximum height, in $m$
$l$ is the tank maximum length, in m
$k$ is the dimensionless coefficient to be determined from table 3.3.3 according to the ratio $b / h$. The intermediate values are determined by interpolation.

Table 3.3.3 - Values for coefficient $k$ for calculating free surface correction ${ }^{1}$

$$
k=\frac{\sin \phi}{12}\left(1+\frac{\tan ^{2} \phi}{2}\right) b / h
$$

where $\cot \phi \leq b / h$

$$
\begin{aligned}
k= & \frac{\cos \phi}{8}\left(1+\frac{\tan \phi}{b / h}\right) \\
& -\frac{\cos \phi}{12(b / h)^{2}}\left(1+\frac{\cot ^{2} \phi}{2}\right)
\end{aligned}
$$

where $\cot \phi>b / h$

[^0]

Figure 8: Effect of tank breadth:height ratio $(b / h)$ on effective FSM ( kg m ), for a half-filled, 100 kg total capacity, rectangular tank.

Now assuming a rectangular tank, for which $\delta=1.0$, and that the tank is half-full (i.e. $v=2 \nabla_{t}$ ), the IMO equation for $M_{\mathrm{fs}}$ (quoted above) may be rearranged as follows:

Condition A: $\cot \phi \geq b / h$

$$
\begin{equation*}
M_{\mathrm{fs}}=\frac{v_{t} b^{2} \rho_{t}}{6 h}\left(1+\frac{\tan ^{2} \phi}{2}\right) \sin \phi \tag{37}
\end{equation*}
$$

Condition B: $\cot \phi<b / h$

$$
\begin{align*}
M_{\mathrm{fs}}= & v_{t} \rho_{t}\left[\frac{h}{4}+\frac{b}{4 \tan \phi}\right. \\
& \left.-\frac{h^{2}}{6 b \tan \phi}\left(1+\frac{1}{2 \tan ^{2} \phi}\right)\right] \sin \phi \tag{38}
\end{align*}
$$

Comparison of these equations (Equations 37 and 38) with the equations derived in $\S 2.1$ and 2.3 (Equations 18 and 35) shows that the equations are similar except for the $\sin \phi$ term. This implies that $M_{\mathrm{fs}}$ is not in fact the free surface moment but the reduction in ship righting moment due to the tank fluid free surface. It should also be noted that Equation 38 is not valid for all angles where $\cot \phi<b / h$, but is limited by the range as described by Equation 3.

It is worth pointing out that if the equations in [1, §3.3.8] are used verbatim to calculate the FSM, the FSM for the vessel in the upright condition will be zero resulting in no reduction to GM.

## 5 CONCLUSIONS

The effect of fluid movement in slack tanks on static stability has been derived from first principles for tanks with rectangular cross-section, half-full of fluid. These equations have been used to verify the static simulation of
fluid movement in Formation Design Systems' hydrostatics program Hydromax.

Numerical experiments with Hydromax indicated that the half-full condition was the worst condition for a cubic tank. The $20 \%$ and $80 \%$ full conditions were reasonably close to the FSM calculated from the upright tank free surface; below $20 \%$ and above $80 \%$ full, the effective FSM was generally less than upright and for tanks between $20 \%$ and $80 \%$ full, the effective FSM was generally greater than upright.

An investigation into the effect of tank breadth:height ratio $(b / h)$ has also been made. Some interesting results were found for tall, narrow tanks $b / h<1.0$, the effective FSM was found to increase dramatically as the heel angle passes through $90^{\circ}$. For broad tanks, $b / h \geq 3.0$, the effective FSM reduces rapidly compared with the upright FSM at heel angles above $30^{\circ}$.

A comparison with the IMO treatment of fluid free surfaces in slack tanks[1, 2] has been made. This indicated that there is a typographical error in $[1, \S 3.3 .3]$ and [2, $\S 3.3 .8]$, these documents state that: " $M_{\mathrm{fs}}$ is the free surface moment at any inclination, in $m$ tonnes", however, examination of the equations provided in these documents indicate that $M_{\mathrm{fs}}$ is in fact the reduction in GZ due to the free surface of the tank in question.

It has been shown that, even for the relatively simple case of a half-filled, rectangular cross-section tank, the FSM is far from constant as the vessel heels. Under many conditions, the FSM varies dramatically from the FSM in the upright condition. For these reasons and given the reality that virtually all vessel hydrostatics are calculated by computer, it is recommended that calculation of a vessel's large angle, static stability should accurately account for the effects of fluid free surfaces in tanks by modelling the actual position of the fluid in the tank at any arbitrary angle of heel and trim rather than approximating this effect
from the tank free surface in the upright condition. It is acknowledged that this practice is accepted by some authorities including IMO[2, §3.3.7.2.1].

## References

[1] Code on Intact Stability for all types of ships covered by IMO instruments. Resolution A.749(18). IMO, 1995.
[2] Amendments to the Code on Intact Stability for all types of ships covered by IMO instruments. Resolution MSC.75(69). IMO, 1999
[3] Hydromax manual. Formation Design Systems, 2004. http://www.formsys.com.


[^0]:    ${ }^{1}$ Only the equations for $k$ are reproduced here, not the entire table.

